Study guide for Ph.D. Examination in Complex Analysis (Math 5120)

Holomorphic (analytic) functions:

(1) Statement of the Jordan curve theorem and the notion of simple rectifiable curves.
(2) The Riemann sphere.
(3) The Cauchy-Riemann equations.
(4) Power series and the disk of convergence.
(5) Linear fractional (Möbius) transformations and conformal mappings.

Integration theory:

(1) Integration along simple rectifiable curves.
(2) The Cauchy-Goursat theorem.
(3) The Cauchy integral formula, Cauchy's estimate.
(4) Morera's theorem and the maximum principle.
(5) The Argument Principle, winding numbers and Rouche's theorem.
(6) The residue theorem and its use in evaluating real-valued integrals.

Representation Theorems:

(1) Taylor and Laurent series.
(2) The maximum modulus theorem, Liouville's theorem, and the fundamental theorem of algebra.
(3) Singularities and their classification.

Harmonic functions:

(1) The mean value theorem, the maximum principle.
(2) Their relation to holomorphic (i.e. complex analytic) functions.
(3) Harmonic conjugates.

Miscellaneous:

(1) The inverse function theorem.
(2) The Schwarz Lemma.
(3) The Schwarz reflection principle.
(4) Normal families.
(5) The Riemann mapping theorem.

References

[1] Lars V. Ahlfors, *Complex analysis*, 3rd edition, An introduction to the theory of analytic functions of one complex variable; International Series in Pure and Applied Mathematics, McGraw-Hill Book Co., New York, 1978, xi+331.

[2] Henri Cartan, *Elementary theory of analytic functions of one or several complex variables*, Éditions Scientifiques Hermann, Paris, 1963.

[3] John B. Conway, *Functions of one complex variable*, Graduate Texts in Mathematics, **11**, 2nd edition, Springer-Verlag, New York, 1978, xiii+317.

[4] Rolf Nevanlinna, Veikko Paatero, *Introduction to complex analysis*, 2nd edition, Translated from the 1965 German original by T. Kövari and G. S. Goodman, AMS Chelsea Publishing, Providence, RI, 2007, x+350.

Additional references

Einar Hille, *Analytic function theory. Vols. 1 and 2*, Introduction to Higher Mathematics, Ginn and Company, Boston, 1959, xi+308.

Konrad Knopp, *Elements of the Theory of Functions*, Vols. I and II Dover Publications, new printing, 320 pp.

Konrad Knopp, *Problem Book* Vols. I and II, Dover Publications, new printing, 272 pp.

A. I. Markushevich, *Theory of functions of a complex variable. Vol. I, II, III*, Second English edition, translated and edited by Richard A. Silverman, Chelsea Publishing Co., New York, 1977, xxii+1238 pp. (three volumes in one; not consecutively paged).

Reinhold Remmert, *Theory of complex functions*, Graduate Texts in Mathematics, **122**,Readings in Mathematics, Springer, 1991, xx+453.

Walter Rudin, *Real and complex analysis*, 3rd edition,McGraw-Hill Book Co., New York, 1987, xiv+416.