Study Guide for Ph.D. Exam in Actuarial Probability

April 28, 2016

Random variables: σ-algebra, probability space, distributions, and independence. Borel-Cantelli lemma, Kolmogorov's zero-one law.

Inequalities: Markov's, Chebychev's, Cauchy-Schwarz, and Jensen's.

Convergence: convergence of random variables almost surely and in probability, weak convergence of probability measures.

Laws of large numbers: weak laws of large numbers, strong law of large numbers.

Central limit theorems: classical, Lindeberg, infinitely divisible laws, characteristic functions.

Decomposition of probability laws: Radon-Nikodym theorem

Conditional probabilities and expectations: existence and basic properties.

Martingales (and sub-martingales and super-martingales, discrete): Doob decomposition, optional stopping, maximal inequality, (sub-)martingale convergence theorem.

Random walk and Brownian motion: basic properties.

References:

- Jeffrey S. Rosenthal, *First Look at Rigorous Probability Theory*, World Scientific Publishing Company; 2nd edition, 2006.