Graduate Research Forum (2024-2025)
Department of Mathematics
University of Connecticut

Sponsored by Mathematical Science Research Collaboratory and Graduate Program

January 25, 2025, Monteith 104

Organizing Committee: Vasileios Chousionis, Lan-Hsuan Huang, Álvaro Lozano-Robledo, Guozhen Lu, Xiaodong Yan

Schedule	
9:00-9:20am	Coffee and Social
9:20-9:30am	Opening remarks
9:30-10:00am	Java Darleen Villano
10:00-10:30am	Linli Shi
10:30-11:00am	Erik Wendt
11:00-11:30pm	Coffee break and Social
11:30-12:00pm	Benjamin York
12:00-12:30pm	Ryan Schroeder
12:30-1:30pm	Working Lunch Break
1:30-1:50pm	Anh Do
1:50-2:10pm	Andrew Delapo
2:10-2:30pm	Zack Boone
2:30-3:00pm	Coffee break and Social
3:00-3:20pm	Michael Albert
3:20-3:40pm	Xiaohang Ma
3:40-4:00pm	Dylan Costa
4:00-5:00pm	Discussions, Social and Working Dinner

Thank you for your participation!

Speaker: Michael Albert

Title: Length Minimizers on Grushin Spaces

Abstract: The analysis of sub-Riemannian spaces grew out of problems related to control theory in the second half of the 20th century. Broadly speaking, these are minimization problems among an admissible class of curves connecting points on a manifold. In the language of metric geometry, the minimizers are called geodesics. Geodesics must satisfy the Hamiltonian equations, an ODE on the cotangent bundle, so geodesics can often be found explicitly in certain models. However, the solutions to the Hamiltonian equations may not be minimizing for their whole duration. Finding this minimization time is non-trivial. We build on the work of Borza (2022) and Agrachev et. al (2019) and study the problem of length minimization on higher dimensional *alpha*-Grushin spaces, a generalization of the Grushin plane, which is one of the most celebrated models in the field of sub-Riemannian geometry.

Speaker: Zack Boone

Title: Removable Sets in Carnot Groups

Abstract: Riemann's removable singularity theorem gives a characterization on when a function, that is only initially assumed to be holomorphic on a punctured domain, can be holomorphically extended to the whole domain. In a similar fashion to removable singularities, a removable set for a space of functions is one in which the functions satisfy some PDE on a domain removed the set, but can be extended to satisfy the PDE on the whole domain. We will give characterizations of removable sets in the context of Carnot groups, specifically focusing on when the space is functions of bounded mean oscillation (BMO) and a generalization of BMO functions, which is the Campanato space.

Speaker: Dylan Costa

Title: Rank Growth of E(K) over Quadratic Extensions

Abstract: A key question in the study of elliptic curves is the determination of the rank of the group of K-rational points, denoted E(K), which measures the number of independent points of infinite order. In this talk, we investigate how the rank of E(K) behaves when K is a quadratic extension of the rationals, exploring concrete examples and general results. Specifically, this talk will explore how the rank of E(K) can increase by 0, 1, or 2 when passing to such extensions as well as how often such an increase might occur. The talk will also touch on the deeper conjectural aspects of this behavior, including connections to the parity conjecture, and will highlight the intricate nature of elliptic curves and the ongoing

research in understanding the rank of E(K) in various field extensions.

Speaker: Andrew Delapo

Title: Computable Categoricity and CSC Spaces

Abstract: In computable structure theory, a computable structure A is said to be "computably categorical" if whenever a computable structure B is isomorphic to A, a computable isomorphism exists between A and B. This has been studied for many types of combinatorial and algebraic structures. In this talk, I will introduce how questions of computable categoricity can be phrased for certain types of countable, second-countable (CSC) topological spaces. In this context, we are interested in the existence or non-existence of certain computable homeomorphisms between two homeomorphic CSC spaces. We will investigate this question for the indiscrete, discrete, and initial segment topologies on the natural numbers. The relevant computability background will be introduced.

Speaker: Anh Do

Title: Introduction to the stability of Caffarelli-Kohn-Nirenberg inequalities in some geometric settings

Abstract: This talk provides an overview of recent advancements in the study of stability results for functional and geometric inequalities, with a particular focus on the Caffarelli-Kohn-Nirenberg (CKN) inequalities. A key component of deriving such stability results is the establishment of corresponding identities and Poincaré inequalities for Gaussian measures. While Poincaré inequalities for Gaussian measures are well-understood in Euclidean spaces, their counterparts in other settings remain unknown. In this talk, we introduce a Poincaré inequality for Gaussian measures on hyperbolic space, along with several variants that incorporate scaling parameters. These results allow us to establish L^2-stability results for a specific case of the Caffarelli-Kohn-Nirenberg inequalities in the hyperbolic setting. This work is part of a joint collaboration with Guozhen Lu, Nguyen Lam, Debdip Ganguly, and Joshua Flynn.

Speaker: Xiaohang Ma

Title: On the Asymptotic Optimality of Variational Path-wise Filter for Two-time-scale Markovian Switching Systems

Abstract: We address the nonlinear filtering problem for two-time-scale Markov switching diffusion systems, reformulating it as a variational problem in stochastic control. Leveraging

the fast-varying nature of the Markov jump process, the system converges to an averaged limit. Additionally, the controlled term in the Markov chain can be omitted, simplifying the variational formulation. The key result shows that, as the small parameter approaches zero, the optimal value of this simplified problem converges to the same value as the limiting variational problem of the original formulation. This establishes the asymptotic equivalence and optimality of the simplified approach. The proofs utilize the variational representation of the nonlinear filtering problem and weak convergence techniques in stochastic control.

Speaker: Ryan Schroeder

Title: Toward Classifying Grin Quiver Representations

Abstract: Let A be an algebra over a field k. An A-module is said to be grin if each submodule is completely determined by its dimension vector. In this talk, we give some description and classification results from the perspective of quiver representations, as well as briefly discuss relationships with cluster algebras and projective geometry.

Speaker: Linli Shi

Title: On higher regulators of Picard modular surfaces

Abstract: The Birch and Swinnerton-Dyer conjecture relates the leading coefficient of the L-function of an elliptic curve at its central critical point to global arithmetic invariants of the elliptic curve. Beilinson's conjectures generalize the BSD conjecture to formulas for values of motivic L-functions at non-critical points. In this talk, I will relate motivic cohomology classes, with non-trivial coefficients, of Picard modular surfaces to a non-critical value of the motivic L-function of certain automorphic representations of the group GU(2,1).

Speaker: Java Darleen Villano

Title: Relativizing computable categoricity

Abstract: In computable structure theory, a computable structure $\mathcal{A}$ is computably categorical if for every computable copy $\mathcal{B}$ of $\mathcal{A}$, there exists a computable isomorphism $f : \mathcal{A} \to \mathcal{B}$. We can relativize this notion to a Turing degree $\mathbf{d}$ by saying that a computable structure $\mathcal{A}$ is computably categorical relative to $\mathbf{d}$ if for every $\mathbf{d}$-computable copy $\mathcal{B}$, there is a $\mathbf{d}$-computable isomorphism between $\mathcal{A}$ and $\mathcal{B}$. In this talk, we compare this relativization to older notions in the literature and discuss its behavior, starting with a 2021 result by Downey, Harrison-Trainor, and Melnikov which shows that this relativized notion behaves poorly in the computably enumerable (c.e.) degrees. In particular, we extend this result

to partial orders of c.e. degrees, study its behavior relative to degrees which are not c.e., and observe for which classes of structures can an example exist which witnesses the poor behavior of this newer relativization of categoricity.

Speaker: Erik Wendt

Title: Rigorous Dimension Estimation for Conformal Fractals

Abstract: Conformal fractals are relevant to many areas of mathematics, and estimates on their Hausdorff dimension are used in number theory (e.g., Zaremba's Conjecture), hyperbolic geometry (e.g., Patterson Sullivan Theory), and PDEs (e.g., scattering theory on hyperbolic surfaces). It is therefore important to establish general, rigorous numerical algorithms for calculating the Hausdorff dimension of a large class of conformal fractals. In this talk, we present theoretical and computational results for rigorous dimension estimation on conformal fractals, and showcase some methods of our proofs.

Speaker: Benjamin York

Title: Galois Representations Attached to Elliptic Curves

Abstract: In this talk, I will give a brief introduction to the theory of Galois representations attached to elliptic curves, and detail my ongoing work in this area.